Mechanisms of reduction of dispersion in Stable Density-Driven Flows in Heterogeneous Porous Media

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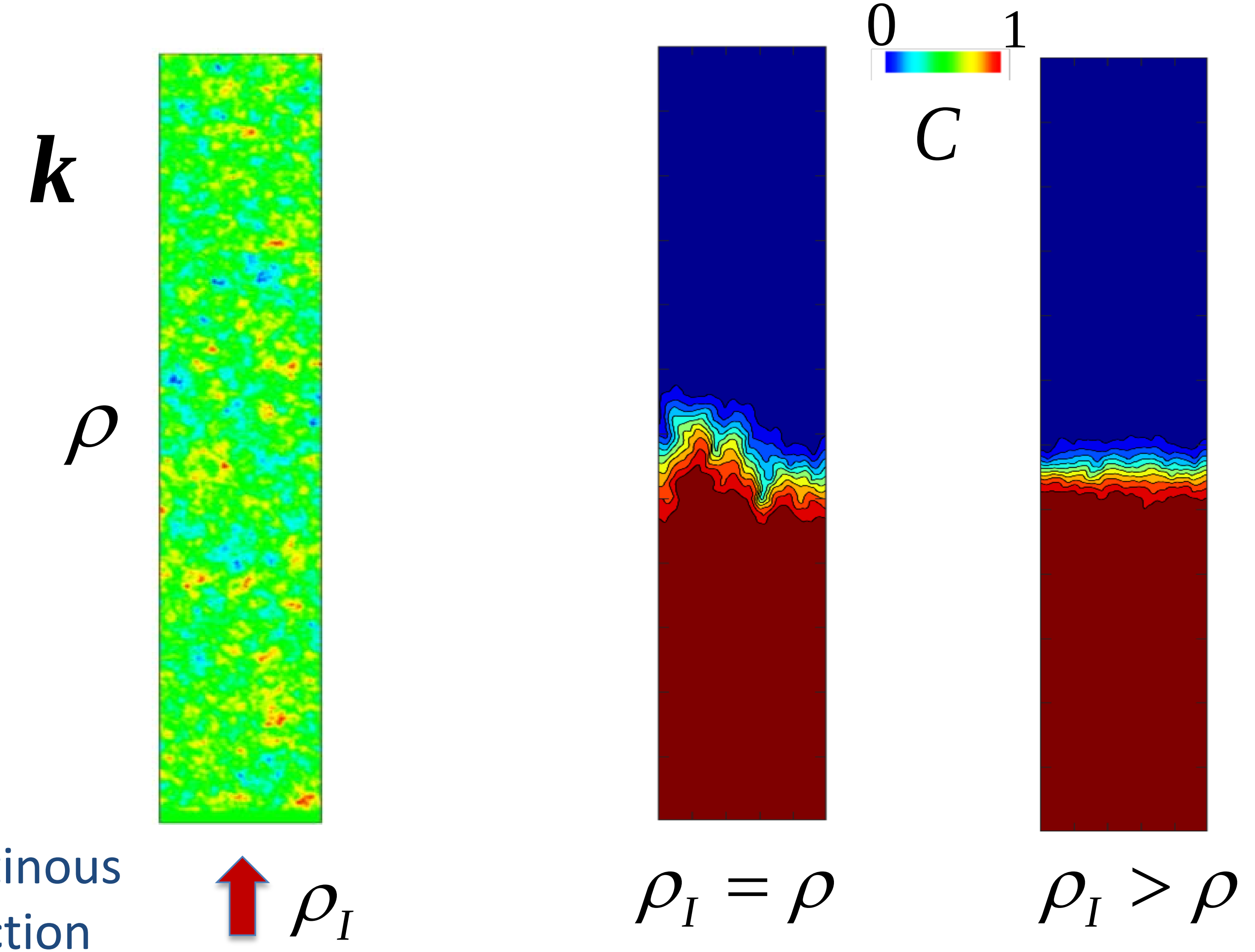
AGU, New Orleans, December, 2017

Stable Variable Density Flow, within **heterogeneous** Porous Media



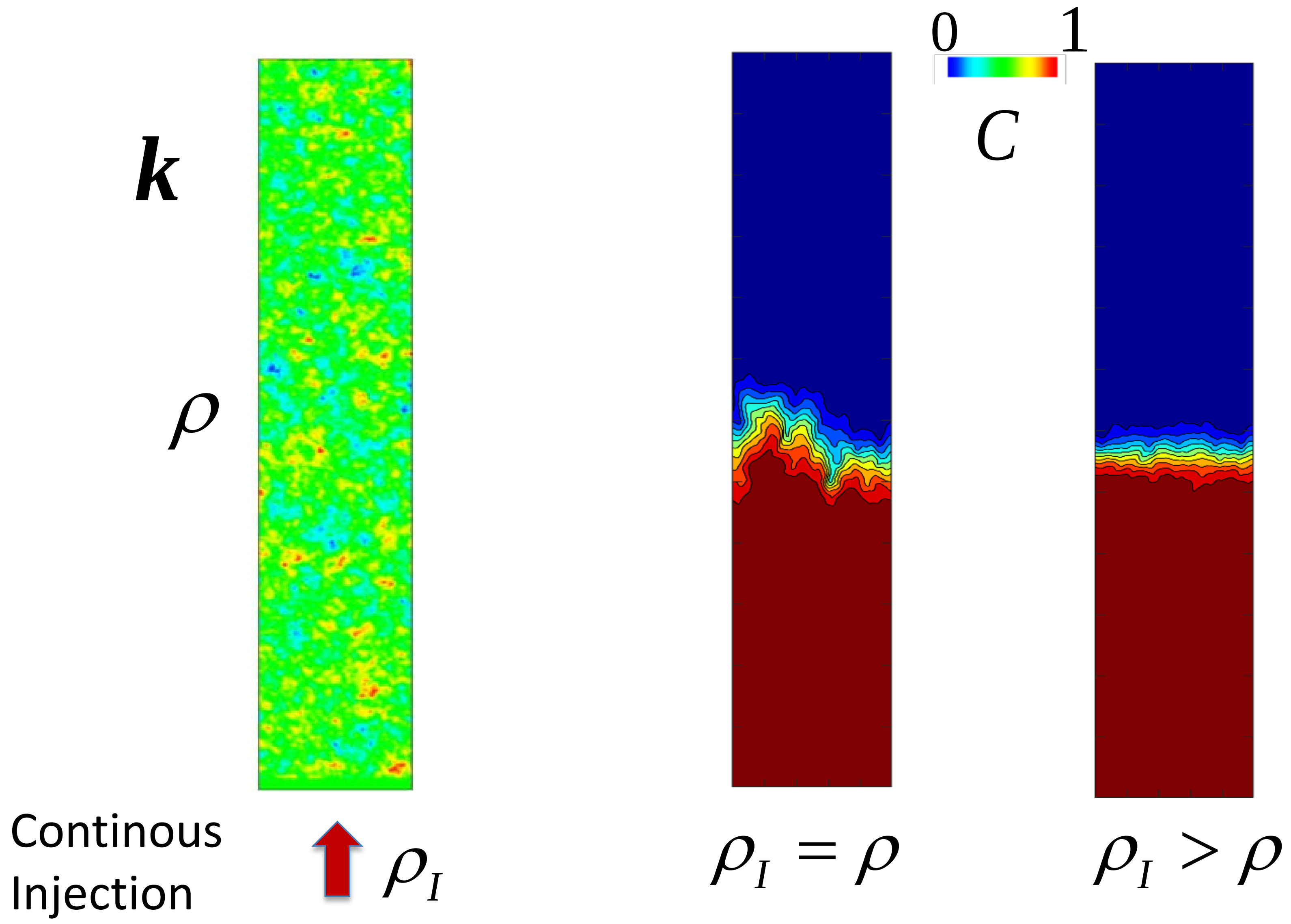
Continous Injection

Settings with variable density are characterized by reduced spreading

Stabilizing effect

Experimental evidences
(*Menand and Woods*, 2005; *Flowers and Hunt*, 2007)

Stable Variable Density Flow, within **heterogeneous** Porous Media



Can we detect the physical mechanisms at the heart of this stabilizing effect?

Interplay between heterogeneity and buoyancy effects.

Stable **Variable** Density Flow, within heterogeneous Porous Media

$$\rho = f(C)$$

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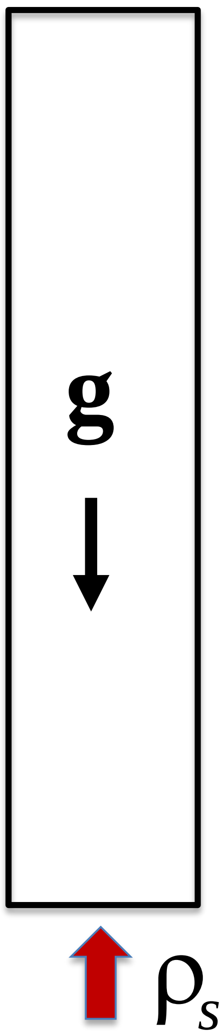

Column Initally saturated
with freshwater

$$\rho_f < \rho_s$$

ρ_s Saltwater: continuos injection

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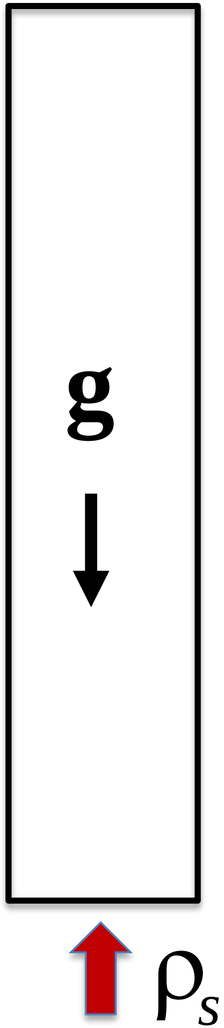

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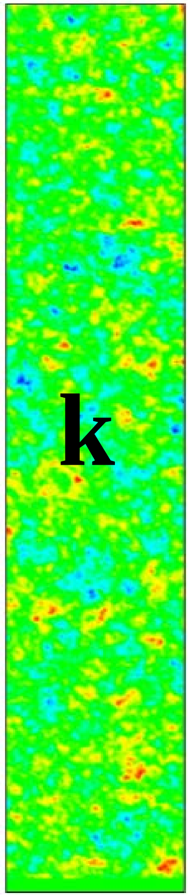
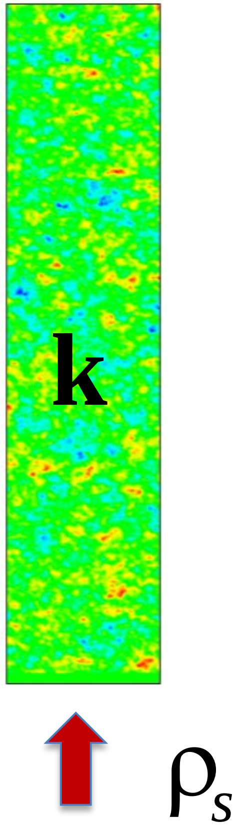


Permeability

Governing Equations (dimensionless)

- Darcy' Law :

$$\boldsymbol{q} = -\boldsymbol{k} \cdot \left(\nabla p + \mathrm{N}_{\mathrm{g}} \rho \nabla z \right) \qquad \boldsymbol{v} = \boldsymbol{q} / \phi$$



$$\mathrm{N}_{\mathrm{g}} = \frac{\Delta\rho \tilde{\mathrm{k}} g}{\mu v_{BC}}$$

Gravity number: relative intensity of buoyancy and viscous forces

$\tilde{\mathrm{k}}$ Geometric mean of k

$$\Delta\rho = \rho_s - \rho_f$$

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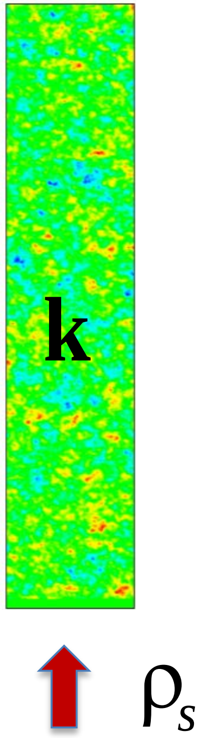
- Mass Conservation: $$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{q}) = 0$$



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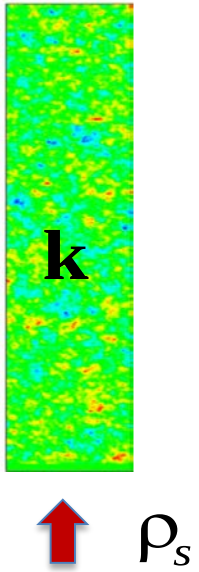
- Mass Conservation: $$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \boldsymbol{q} \right) = 0$$
- Transport Eq: $$\frac{\partial \left(\phi \rho C \right)}{\partial t} + \nabla \cdot \left(\rho \boldsymbol{q} C \right) - \nabla \cdot \left(\phi \rho \boldsymbol{D} \cdot \nabla C \right) = 0$$

$$\boldsymbol{D} = \frac{|\mathbf{v}|}{\mathrm{Pe}} \mathbf{I}; \qquad \mathrm{Pe} = \sqrt{\tilde{\mathrm{k}}} / \alpha$$

Peclet number: ratio between advective and dispersive transport rate



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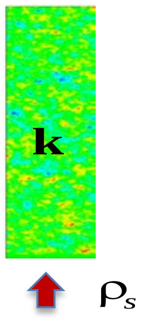
- Constitutive Law: $$\rho = \rho_f + C$$

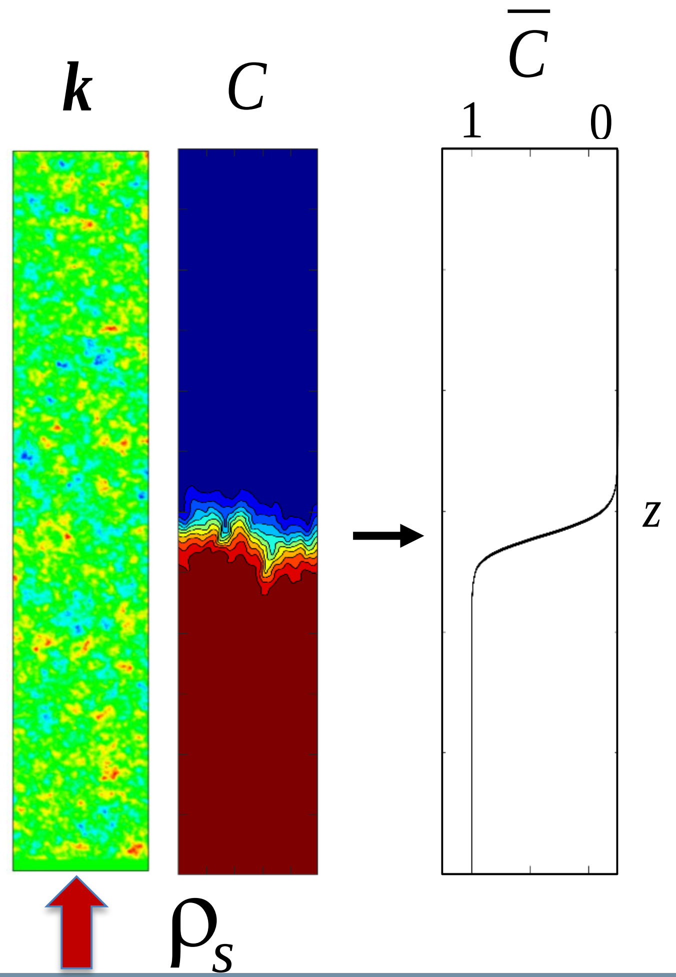
COUPLED



$$\rho = \frac{\rho}{\Delta\rho} \qquad C = \frac{\mathrm{C}}{\mathrm{C_s}}$$

Model

Section Averaged Concentration



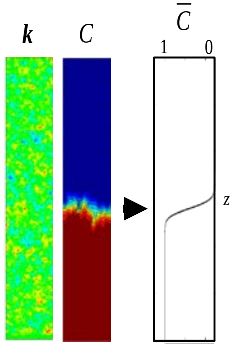
$$C = \overline{C} + C'$$

$$\mathbf{v} = \overline{\mathbf{v}} + \mathbf{v}'$$

…

Method

Decompose the velocity of vector fluctuations as the sum of *stationary* component (i.e. constant density) and a *dynamic* component (i.e. stabilizing buoyancy effects).



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$$\boldsymbol{v}'(y,z,t) = \boldsymbol{v}^{st}{}'(y,z) + \boldsymbol{v}^{dy}{}'(y,z,t)$$

Constant density

$$\boldsymbol{v}^{st}{}' \propto k'$$

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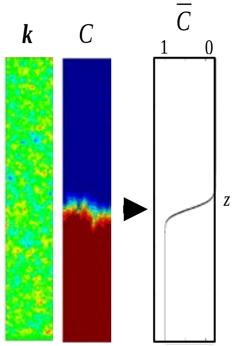

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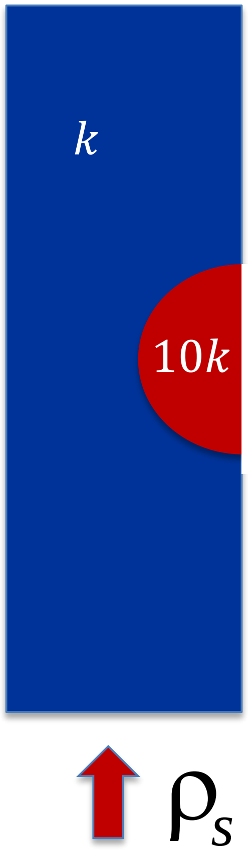
Density variations

$$\mathbf{v}^{dy}{}' \propto -\mathrm{N_g} C'$$

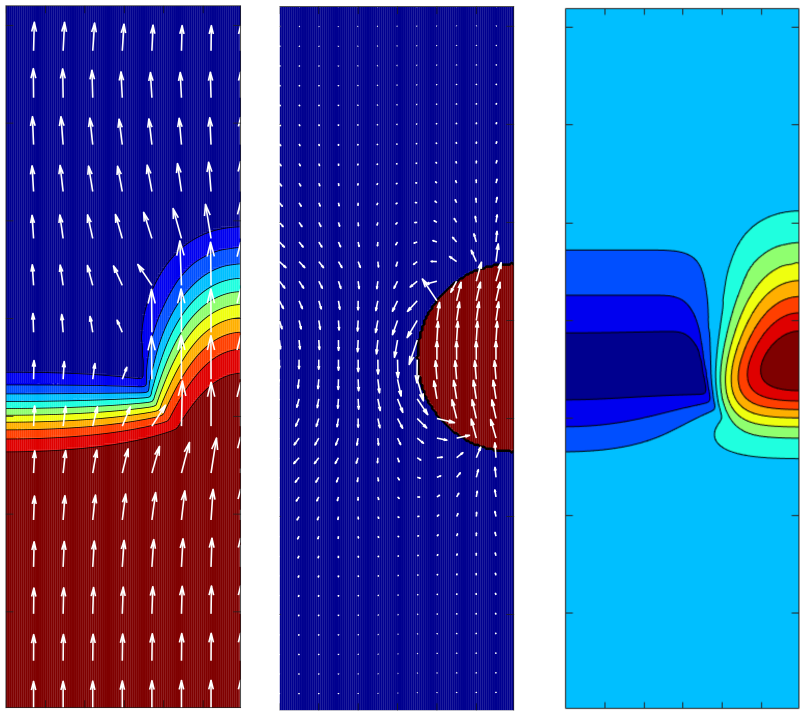
Coupling and **triggering** of the **stabilizing mechanism**



Coupling and **triggering** of the **stabilizing mechanism**

$$\rho = cost$$

C $\quad$ $\boldsymbol{v}^{st}{}' \propto k'$ $\quad$ C'



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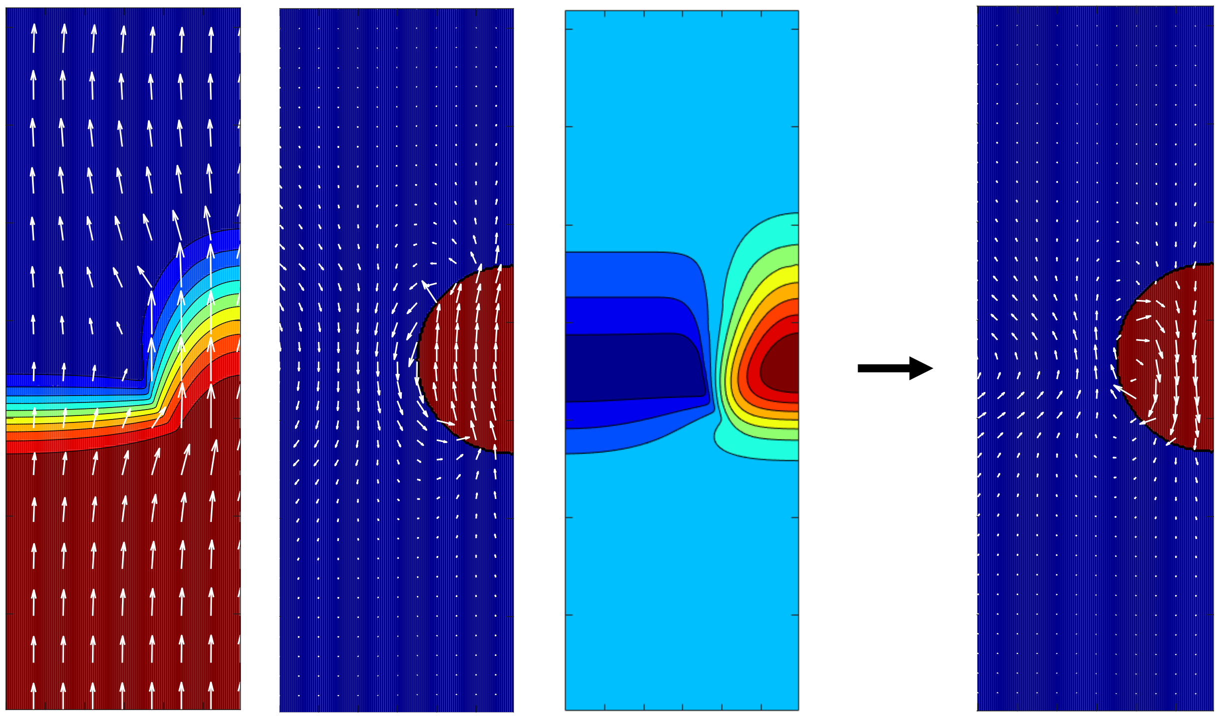

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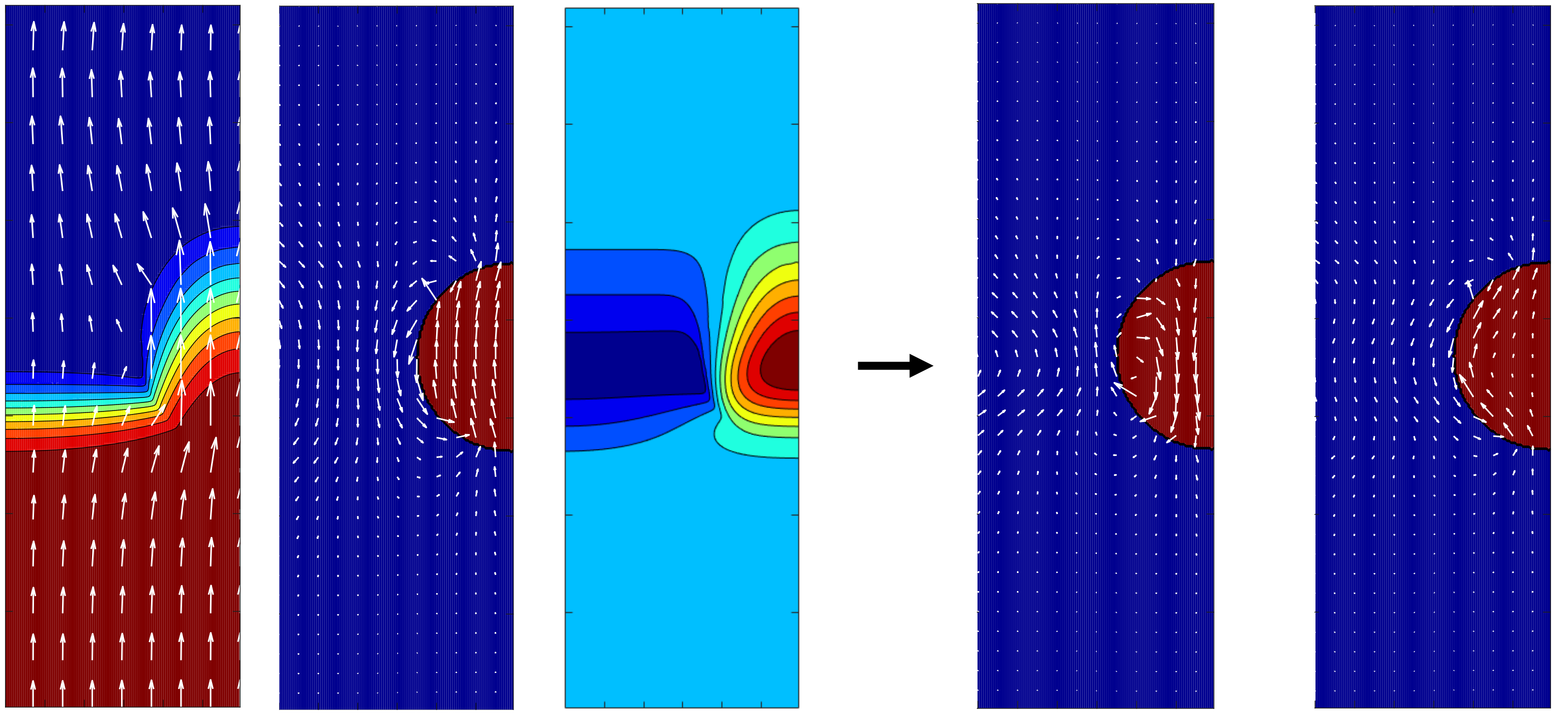
$$C \qquad \mathbf{v}^{st}\,' \propto k' \qquad C' \qquad \mathbf{v}^{dy}\,' \propto -\mathrm{N_g} C' \qquad \mathbf{v}' = \mathbf{v}^{st}\,' + \mathbf{v}^{dy}\,'$$



The **Flow Field regularization** results in **reduction of solute Dispersion**

$\rho = cost$ $\quad\quad$ $\rho \neq cost$

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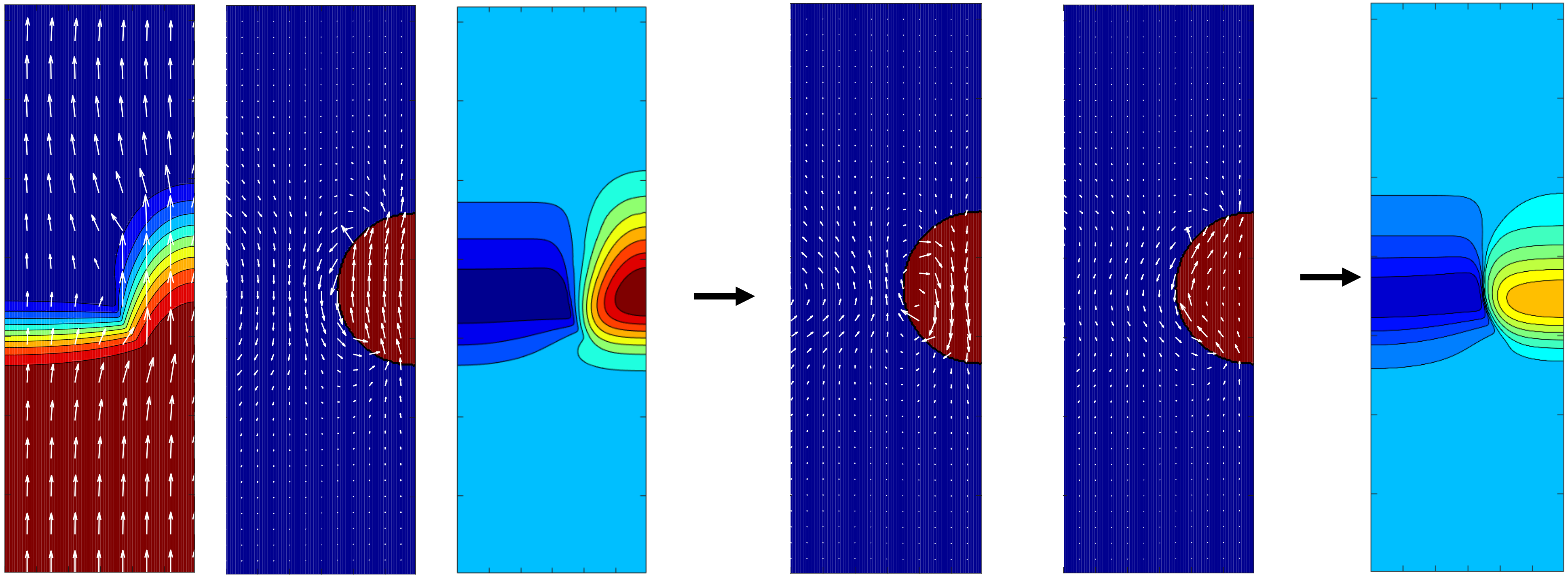

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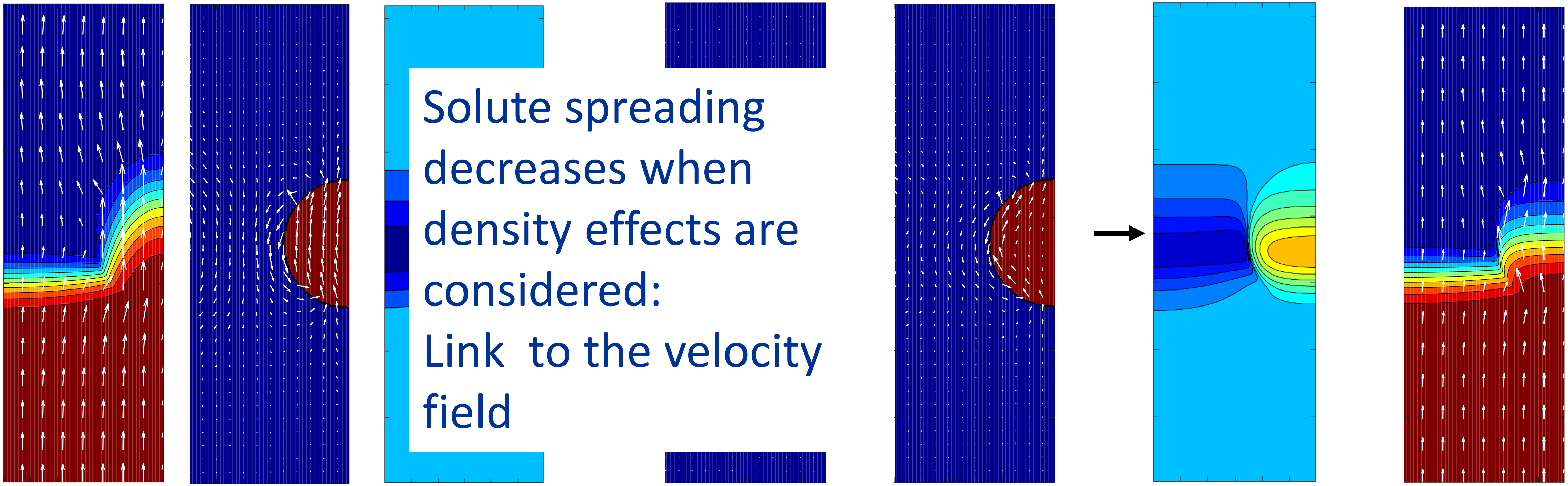
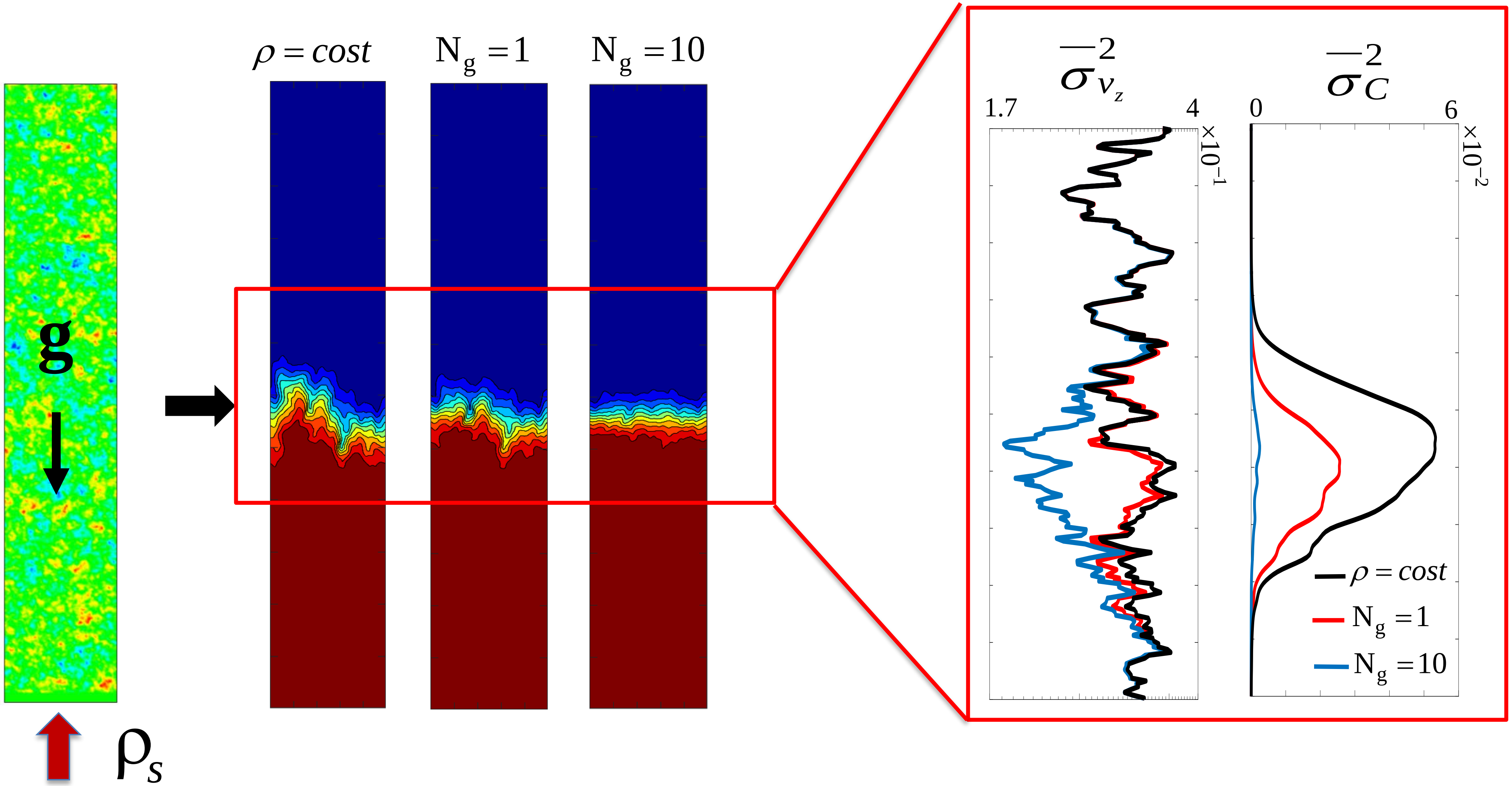


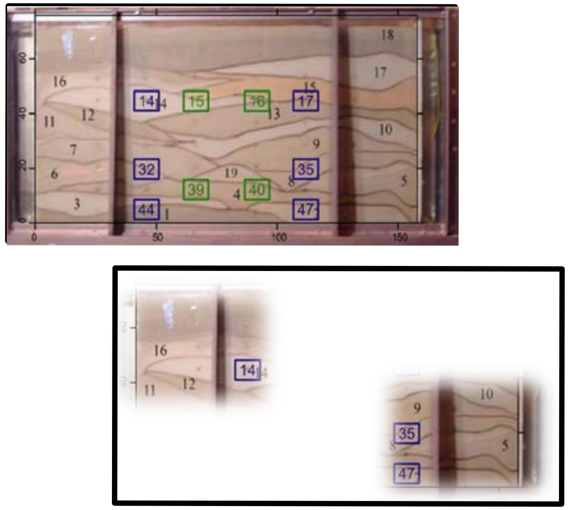
Solute spreading decreases when density effects are considered:
Link to the velocity field

Same mechanism in randomly heterogeneous media



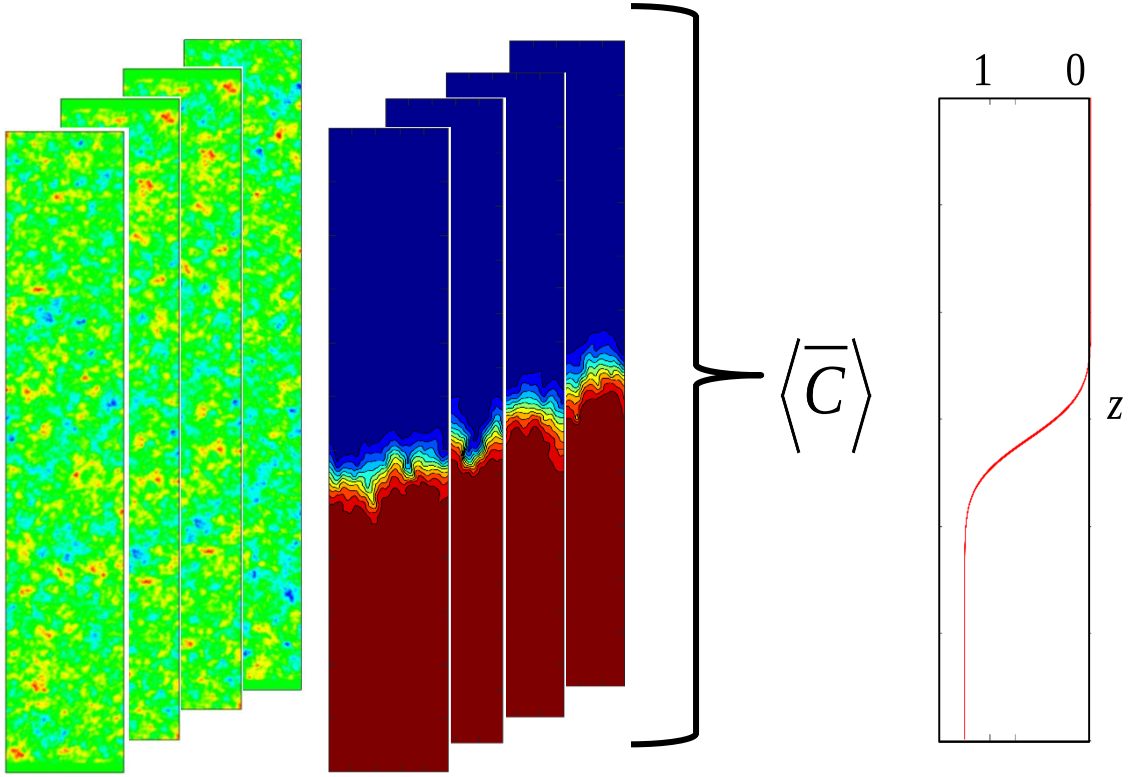
Model

- **Upscaled model**: Section Averaged concentration.
- **Uncertainty**: Ensemble moments.



$$\boldsymbol{k} = \exp(Y(y,z))\mathbf{I}$$

$$C_Y(\boldsymbol{r}) = \sigma_Y^2 \exp\left(-\frac{|\boldsymbol{r}|}{l}\right)$$



Model

Ensemble mean of the Section Averaged concentration

$$\frac{\partial\langle\bar{C}\rangle}{\partial t}+\frac{\partial\langle\bar{C}\rangle}{\partial z}-\frac{1}{\mathrm{Pe}}\frac{\partial^2\langle\bar{C}\rangle}{\partial z^2}+\frac{\partial\left\langle\overline{v_z{}'C'}\right\rangle}{\partial z}=0$$

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Ensemble dispersive flux

- Accommodate the details lost during the Upscaling
- Measure of the solute spreading
- Account for the uncertainty

Model

Ensemble mean of the Section Averaged concentration

$$\frac{\partial \langle \bar{C} \rangle}{\partial t} + \frac{\partial \langle \bar{C} \rangle}{\partial z} - \frac{1}{\mathrm{Pe}} \frac{\partial^2 \langle \bar{C} \rangle}{\partial z^2} + \frac{\partial \langle \overline{v_z' C'} \rangle}{\partial z} = 0$$

Ensemble dispersive flux

$$\langle \overline{v_z' C'} \rangle = -\int_0^t \int_0^H \langle \overline{v_z'(z,t) v_z'(\xi,\tau)} \rangle G_z^T(z,t,\xi,\tau)\, d\xi\, d\tau \frac{\partial \langle \bar{C}(z,t) \rangle}{\partial z}$$

Covariance of Vertical Velocity

$$\boldsymbol{v}' = \boldsymbol{v}^{st}{}' + \boldsymbol{v}^{dy}{}'$$

$$\left\langle \overline{v'_z(z,t)v'_z(\xi,\tau)} \right\rangle = \left\langle \overline{v_z^{st}{}'(z)v_z^{st}{}'(\xi)} \right\rangle + \left\langle \overline{v_z^{st}{}'(z)v_z^{dy}{}'(\xi,\tau)} \right\rangle + \left\langle \overline{v_z^{dy}{}'(z,t)v_z^{st}{}'(\xi)} \right\rangle + \left\langle \overline{v_z^{dy}{}'(z,t)v_z^{dy}{}'(\xi,\tau)} \right\rangle$$

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Promotes Dispersion

Covariance of Vertical Velocity

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Promotes Dispersion

Buoyant stabilizing Actions!!

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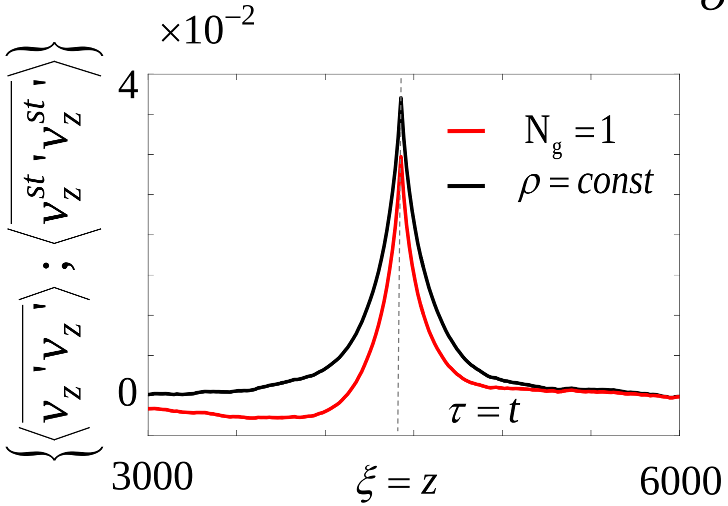
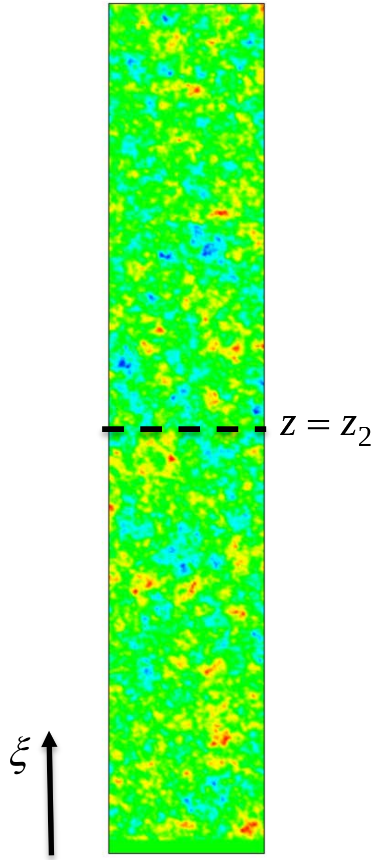
$z = z_2$

$t = t_2$

$\sigma_Y^2 = 0.1$





- Reduction of the velocity variance (i.e. peaks of the covariance) and of the correlation scale of the flow field, for the variable density case compared with the constant density scenario.

Covariance of Vertical Velocity

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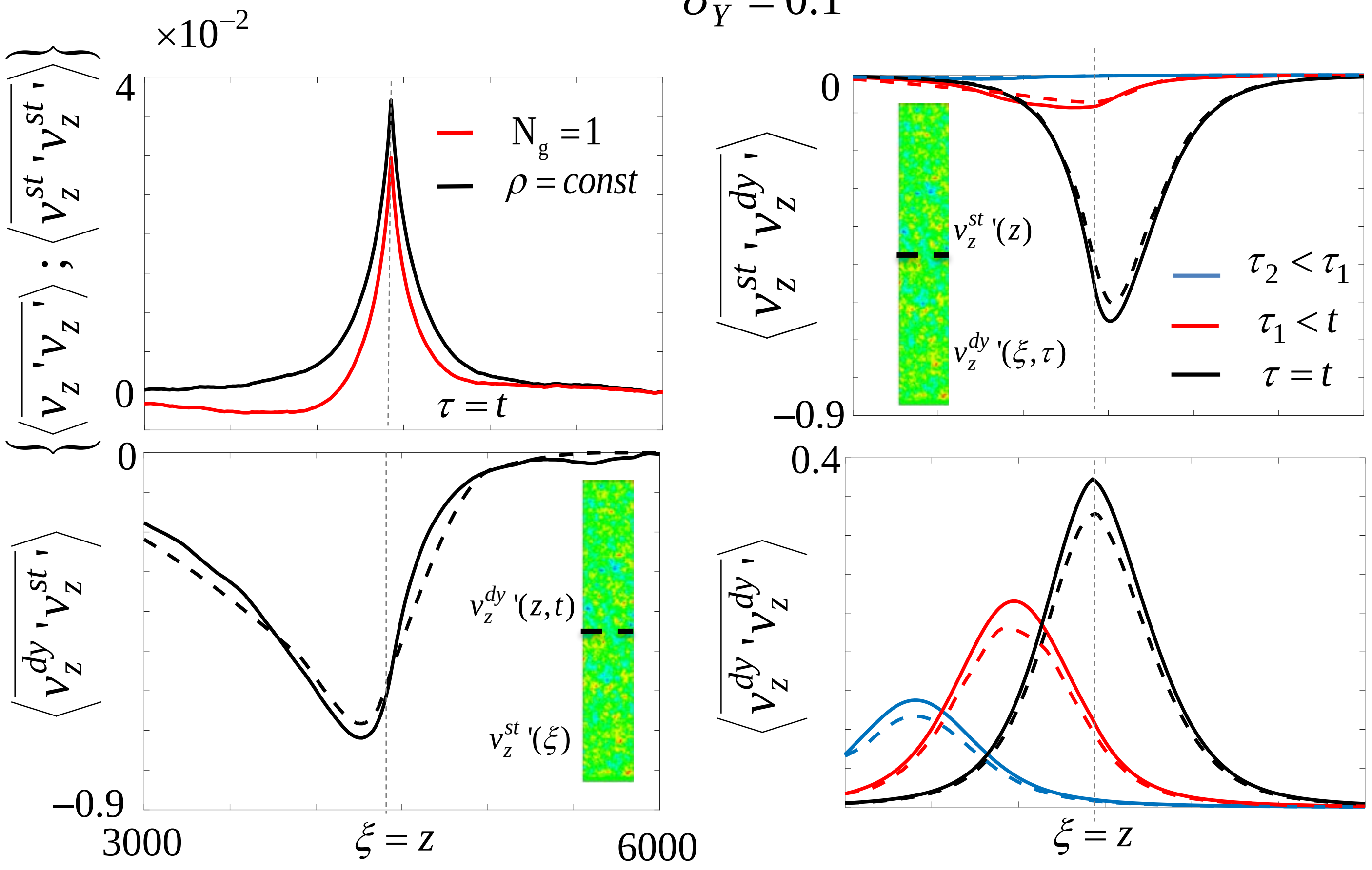
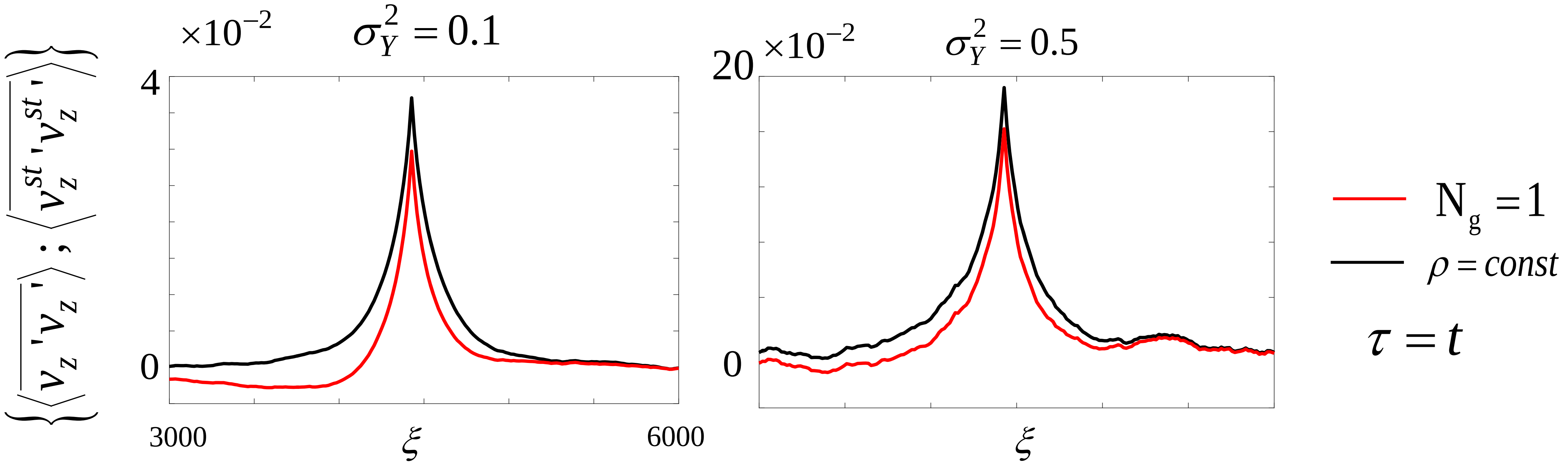


- Reduction of the velocity variance (i.e. peaks of the covariance) and of the correlation scale of the flow field, for the variable density case compared with the constant density scenario.

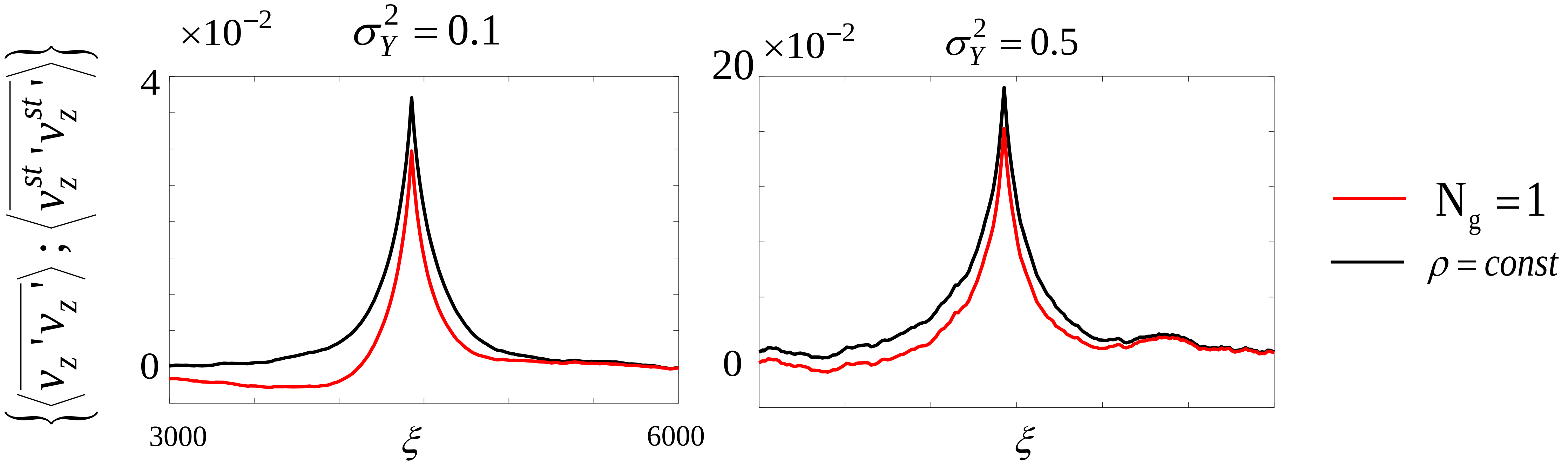
- The *stationary-dynamic cross-*covariances are negative!

Covariance of Vertical Velocity



The relative importance of stabilizing actions decreases as the heterogeneity increases!

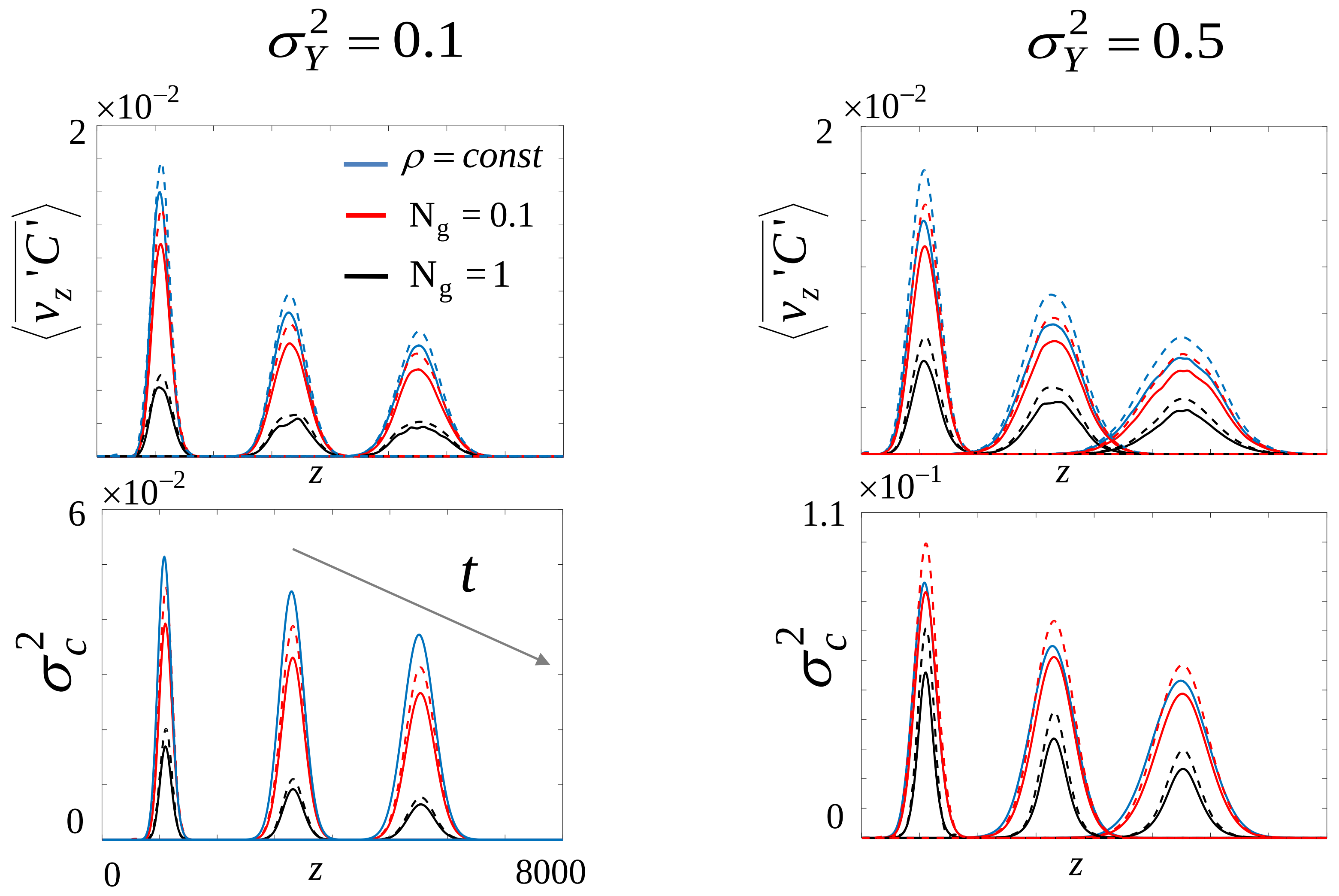
Covariance of Vertical Velocity



The relative importance of stabilizing actions decreases as the heterogeneity increases!

$$\left\langle \overline{v_z^{st\,\prime} v_z^{st\,\prime}} \right\rangle \propto \sigma_Y^2 \qquad \left\langle \overline{v_z^{st\,\prime} v_z^{dy\,\prime}} \right\rangle \propto -N_g \sigma_Y$$

Reduction of **ensemble dispersive flux and concentration variance**



Solute spread and concentration variance decrease as the stabilizing buoyant effects increase.

Stabilizing effect decreases as heterogeneity increases.

Conclusions

- We analyze the occurrence of **solute spreading reduction** in randomly heterogeneous porous media for a density-driven stable flow setting. Interaction between **heterogeneity** and **buoyancy** (stabilizing) effects.

- Velocity is decomposed in terms of **stationary and dynamic** components.

- Solute dispersion reduction is linked to the emergence of **negative cross-covariances between stationary and dynamic velocity fluctuations**.

- The relative importance of the stabilizing components tends **to decrease as the heterogeneity of the porous medium increases.**

Thank you

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